Calc BC : 9-4 Pg 511 # 7, 12, 15, 23, 31, 36, 37, 40, 43

⑦ $\sum_{n=0}^{\infty} x^n$; geometric so

converges for $|x|<1$.

$R=1$

⑫ $\sum_{n=0}^{\infty} \frac{n\cdot x^n}{n+2}$

$\lim_{n\to\infty} \frac{(n+1)|x|^{n+1}}{n+3} \cdot \frac{n+2}{n|x|^n}$

$= \lim_{n\to\infty} \frac{(n+1)(n+2)}{n(n+3)} \cdot |x|$

$L = |x|$ converges for

$|x|<1$, $R=1$

⑮ $\sum_{n=0}^{\infty} \frac{n(x+3)^n}{5^n}$

$L = \lim_{n\to\infty} \frac{(n+1)|x+3|^{n+1}}{5^{n+1}} \cdot \frac{5^n}{n\cdot|x+3|^n}$

$= \lim_{n\to\infty} \frac{n+1}{n} \cdot \frac{|x+3|}{5}$

$L = \frac{|x+3|}{5} < 1$

$|x+3| < 5$

$R=5$

㉓ $\sum_{n=0}^{\infty} \frac{(x-1)^{2n}}{4^n} = \sum_{n=0}^{\infty} \left(\frac{(x-1)^2}{4}\right)^n$

geometric with $r = \frac{(x-1)^2}{4}$

need $\left|\frac{(x-1)^2}{4}\right| < 1$

$|(x-1)^2| < 4$

$|x-1| < 2 \Rightarrow -1 < x < 3$

$S_n = \frac{a}{1-r}$; $\frac{1}{1-\frac{(x-1)^2}{4}} = S_n$

$\frac{4}{4-(x-1)^2} = \frac{-4}{x^2-2x-3} = S_n$

㉛ $\sum_{n=1}^{\infty} \frac{n^2-1}{2^n}$

$\lim_{n\to\infty} \frac{(n+1)^2-1}{2^{n+1}} \cdot \frac{2^n}{n^2+1}$

$= \lim_{n\to\infty} \frac{1}{2} \cdot \frac{n^2+2n}{n^2+1} = \frac{1}{2} < 1$

converges by

Ratio Test.

(36) $\displaystyle\sum_{n=0}^{\infty} \frac{n^{10}}{10^n}$

$$L = \lim_{n\to\infty} \frac{(n+1)^{10}}{10^{n+1}} \cdot \frac{10^n}{n^{10}}$$

$$= \frac{1}{10} < 1$$ Converges by Ratio Test

(37) $\displaystyle\sum_{n=1}^{\infty} \frac{(n+3)!}{3!\,n!\,3^n}$

$$L = \lim_{n\to\infty} \frac{(n+4)!}{3!\,(n+1)!\,3^{n+1}} \cdot \frac{3!\,n!\,3^n}{(n+3)!}$$

$$= \lim_{n\to\infty} \frac{n+4}{3\cdot(n+1)} = \frac{1}{3} < 1$$

Converges by Ratio Test

(40) $\displaystyle\sum_{n=1}^{\infty} n!\,e^{-n}$

$$\lim_{n\to\infty} \frac{n!}{e^n} = \infty$$

Diverges by n^{th} term test.

(Could also use Ratio Test)

(43) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{(2n+1)!}$

$$L = \lim_{n\to\infty} \frac{(n+1)!}{(2(n+1)+1)!} \cdot \frac{(2n+1)!}{n!}$$

$$= \lim_{n\to\infty} \frac{n+1}{(2n+3)(2n+2)} = 0 < 1$$

Converges by Ratio Test